1. Objectives of the work

The incompressible Navier-Stokes equations

\[ \begin{align*}
\rho \frac{D}{Dt} \mathbf{u} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) & = -\nabla p + \nabla \cdot \mathbf{T}, \\
\frac{D}{Dt} \rho & = 0,
\end{align*} \]

with homogeneous Dirichlet b.c.: \( \mathbf{u} = 0 \) on \( \partial \Omega \).

The standard Marker-And-Cell scheme \[5\]

\[ \begin{align*}
\mathbf{u}^{n+1} = \mathbf{u}^n + \frac{\Delta t}{\rho} (\mathbf{u}^{n+1} \otimes \mathbf{u}^{n+1}) + \frac{\Delta t}{\rho} \nabla p^{n+1}, \\
\nabla \cdot \mathbf{u}^{n+1} = 0.
\end{align*} \]

Advantages and drawbacks

- minimal number of unknowns,
- no pressure stabilization needed,
- simplicity and robustness of the scheme,
- not adapted for complex geometries.

Convergence analysis

- Shin Shiketekawa, 96, 97,
- Nicolades Wu, 96,
- Blanc, 95

Key points of the generalization of the MAC scheme

- velocity components at mid-edges of the pressure mesh,
- staggered meshes.

Structured grids:

- pressure at the cell-center and normal velocity components at mid-edges,
- non-conforming meshes:

Generalization to unstructured grids

Covolume approaches for non-structured grids: Porsching (98) Hall, Nicolades (90)

Aims here: prove convergence in the case of:

- Time dependent Navier Stokes
- Non conforming meshes
- General meshes

2. The MAC scheme

Discrete unknowns and spaces

- \( (\mathbf{u}, p) \): Cartesian-rectangular mesh of \( \Omega \), mesh size: \( h \),
- \( z \)-edges of \( M \),
- \( \mathbf{u}_n = (u_n, v_n) \) is the normal vector to \( \mathbf{u} \), with \( \mathbf{n} \geq 0 \).

Unknowns for \( \mathbf{p}^n, \mathbf{u}^n, \mathbf{v}^n \in \mathbb{L}^2(\Omega) \), \( \mathbf{n} \geq 0 \), approximate value of \( u_n, v_n \) in \( u_n, v_n \).

Weak formulation of the NS equations

\[ \left[ \begin{array}{c}
(\mathbf{u}, \mathbf{v}) \mathbf{u} = \left( \mathbf{u}^n, \mathbf{v}^n \right) \mathbf{u}^n \mathbf{v}^n
\end{array} \right] \]