POSTER

Asymptotic behaviour of ground states of a quasilinear elliptic equation with a vanishing parameter

Wedad ALBALAWI
Department of Mathematics, Swansea University, Singleton Park, Swansea, SA2 8PP, Wales, United Kingdom.

V. MOROZ
Department of Mathematics, Swansea University, Singleton Park, Swansea, SA2 8PP, Wales, United Kingdom.

C. MERCURI
Department of Mathematics, Swansea University, Singleton Park, Swansea, SA2 8PP, Wales, United Kingdom.

Abstract
We study the asymptotic behaviour of ground state solutions of the quasilinear elliptic equation

$$-\Delta_p u + \varepsilon u^{p-1} - |u|^{q-2}u + |u|^{l-2}u = 0 \quad \text{in} \quad \mathbb{R}^N,$$

where $1 < p < N$, $p < q < l < +\infty$ and $\varepsilon > 0$ is a small parameter. For $\varepsilon \to 0$, we give a complete characterization of all possible asymptotic regimes as a function of parameters $q$, $l$ and $N$. The behaviour of ground state solutions depends on whether $q$ is less than, is equal to or is greater than the critical Sobolev exponent $p^* := \frac{pN}{N-p}$. In the supercritical case $q > p^*$ ground states of $(P_\varepsilon)$ converge to the solution of the limit equation with $\varepsilon = 0$. In the subcritical case $q < p^*$, after a rescaling, ground states of $(P_\varepsilon)$ converge to the solution of the equation in which the last term is absent. In the most delicate critical case $q = p^*$ the asymptotic behaviour of the ground states of $(P_\varepsilon)$ after a rescaling is given by a particular solution of the critical Emden-Fowler equation, but the choice of the associated rescaling depends on the dimension $N$ in a nontrivial way.

Keywords. Critical Sobolev exponent; subcritical; critical and supercritical non-linearity; Pohožaev’s identity; asymptotic behaviour.
References
