Global exact controllability of the bilinear of Schrödinger potential type models on quantum graphs

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Abstract
In quantum mechanics any pure state of a system is mathematically represented by a wave function $\psi$ in the unit sphere of a Hilbert space $H$. The dynamics of a particle constrained in a compact graph type structure $G$ and excited by a controlled field is represented by the Cauchy problem in $H = L^2(\Gamma, \mathbb{C})$

\[
\begin{cases}
i\partial_t \psi(t, x) = A\psi(t, x) + u(t)B\psi(t, x), & x \in G, \ t \in (0, T), \\
\psi(0, x) = \psi^0(x),
\end{cases}
\]  
\quad (1)

The operator $B$ is bounded symmetric, $u$ is a control function and $\psi^0(x)$ is the initial state of the system. The operator $A = -\Delta$ is the Laplacian equipped with self-adjoint type boundary conditions into the vertices of the graph.

The controllability of the bilinear Schrödinger Equation on bounded intervals has been widely studied in the literature starting by the seminal work on bilinear systems of Ball, Mardsen and Slemrod. Although in the last years quantum graphs have been largely used to study complex phenomena, the controllability on such models of the bilinear Schrödinger Equation was not still addressed.

The aim of the work is to study the controllability of (1) for a generic quantum graph. I analyze how the boundary conditions and the structure of the graph affect the controllability. I start by providing the well-posedness of (1) and I proceed by ensuring the global exact controllability. After I show how the previous result implies the contemporaneous controllability. I conclude by introducing a weaker notion of controllability that one can use when the previous techniques fail.

Mots clés. Schrödinger equation, global exact controllability, inversion problems, quantum graphs, moment problems, Beurling Theorem, perturbation theory.

References

