POSTER

Construction of a stable blowup solution with a prescribed behavior for a non-scaling invariant semilinear heat equation

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Abstract

We consider in [1] the semilinear heat equation

\[ \partial_t u = \Delta u + |u|^{p-1}u \ln(\alpha (u^2 + 2)), \]

in the whole space \( \mathbb{R}^n \), where \( p > 1 \) and \( \alpha \in \mathbb{R} \). Unlike the standard case \( \alpha = 0 \), this equation is not scaling invariant. We construct for this equation a solution which blows up in finite time \( T \) only at one blowup point \( a \), according to the following asymptotic dynamics:

\[ u(x,t) \sim \psi(t) \left(1 + \frac{(p-1)|x-a|^2}{4p(T-t) \ln(T-t)}\right)^{-\frac{1}{p-1}} \text{ as } t \to T, \]

where \( \psi(t) \) is the unique positive solution of the ODE

\[ \psi' = \psi^p \ln\alpha (\psi^2 + 2), \quad \lim_{t \to T} \psi(t) = +\infty. \]

As a consequence of the above asymptotic, we can describe the explosion of the solution at time \( T \) around the blowup point \( a \) as follows

\[ u(x,T) \sim \left[ \frac{(p-1)^2|x|^2}{8p \ln|x|} \right]^{-\frac{1}{p-1}} \left( \frac{4|\ln|x||}{p-1} \right)^{-\frac{1}{p-1}} \text{ as } x \to a. \]

The construction relies on the reduction of the problem to a finite dimensional one and a topological argument based on the index theory to get the conclusion. By the interpretation of the parameters of the finite dimensional problem in terms of the blowup time and the blowup point, we show the stability of the constructed solution with respect to perturbations in initial data. To our knowledge, this is the first successful construction for a genuinely non-scale invariant PDE of a stable blowup solution with the derivation of the blowup profile. From this point of view, we consider our result as a breakthrough.

Keywords. blowup solution, blowup profile, non-scaling invariant heat equation.
References